- 1. A particle *P* of mass 0.6 kg is released from rest and slides down a line of greatest slope of a rough plane. The plane is inclined at 30° to the horizontal. When *P* has moved 12 m, its speed is 4 m s^{-1} . Given that friction is the only non-gravitational resistive force acting on *P*, find
 - (a) the work done against friction as the speed of P increases from 0 ms^{-1} to 4 ms^{-1} ,
 - (b) the coefficient of friction between the particle and the plane.

(4) (Total 8 marks)

(4)

2. [In this question i and j are perpendicular unit vectors in a horizontal plane.]

A ball of mass 0.5 kg is moving with velocity $(10\mathbf{i} + 24\mathbf{j}) \text{ ms}^{-1}$ when it is struck by a bat. Immediately after the impact the ball is moving with velocity 20 $\mathbf{i} \text{ ms}^{-1}$.

Find

- (a) the magnitude of the impulse of the bat on the ball,
- (b) the size of the angle between the vector *i* and the impulse exerted by the bat on the ball,

(2)

(4)

(c) the kinetic energy lost by the ball in the impact.

(3) (Total 9 marks) 3.



A ball is projected with speed 40 m s⁻¹ from a point *P* on a cliff above horizontal ground. The point *O* on the ground is vertically below *P* and *OP* is 36 m. The ball is projected at an angle θ° to the horizontal. The point *Q* is the highest point of the path of the ball and is 12 m above the level of *P*. The ball moves freely under gravity and hits the ground at the point *R*, as shown in the diagram above. Find

- (a) the value of θ ,
- (b) the distance *OR*,
- (c) the speed of the ball as it hits the ground at *R*.

(3) (Total 12 marks)

(6)

4.



A particle *P* of mass 2 kg is projected up a rough plane with initial speed 14 m s⁻¹, from a point *X* on the plane, as shown in the diagram above. The particle moves up the plane along the line of greatest slope through *X* and comes to instantaneous rest at the point *Y*. The plane is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{7}{24}$. The coefficient of friction between the particle and the plane is $\frac{1}{8}$.

(a) Use the work-energy principle to show that XY = 25 m.

After reaching *Y*, the particle *P* slides back down the plane.

(b) Find the speed of *P* as it passes through *X*.

(4) (Total 11 marks)

(7)



A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B, where AB = 14 m. At A the package has speed 12 m s⁻¹ and at B the package has speed 8 m s⁻¹, as shown in the diagram above. Find

(a) the total energy lost by the package in travelling from A to B,

(5)

(b) the coefficient of friction between the package and the ramp.

(5) (Total 10 marks)

- 6. A particle *P* has mass 4 kg. It is projected from a point *A* up a line of greatest slope of a rough plane inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between *P* and the plane is $\frac{2}{7}$. The particle comes to rest instantaneously at the point *B* on the plane, where AB = 2.5 m. It then moves back down the plane to *A*.
 - (a) Find the work done by friction as *P* moves from *A* to *B*.

(b) Using the work-energy principle, find the speed with which P is projected from A.

(4)

(4)

(c) Find the speed of P when it returns to A.

(4) (Total 12 marks)

- 7. A brick of mass 3 kg slides in a straight line on a horizontal floor. The brick is modelled as a particle and the floor as a rough plane. The initial speed of the brick is 8 m s⁻¹. The brick is brought to rest after moving 12 m by the constant frictional force between the brick and the floor.
 - (a) Calculate the kinetic energy lost by the brick in coming to rest, stating the units of your answer.

(2)

(b) Calculate the coefficient of friction between the brick and the floor.

(4) (Total 6 marks)

8.



A small package *P* is modelled as a particle of mass 0.6 kg. The package slides down a rough plane from a point *S* to a point *T*, where ST = 12 m. The plane is inclined at an angle of 30° to the horizontal and *ST* is a line of greatest slope of the plane, as shown in the diagram. The speed of *P* at *S* is 10 m s⁻¹ and the speed of *P* at *T* is 9 m s⁻¹. Calculate

- (a) the total loss of energy of P in moving from S to T,
- (b) the coefficient of friction between *P* and the plane.

(4)

(5) (Total 9 marks)

9.	[In this question \mathbf{i} and \mathbf{j} are perpendicular unit vectors in a horizontal plane.]		
	A ball has mass 0.2 kg. It is moving with velocity (30i) m s ⁻¹ when it is struck by a bat. The bat exerts an impulse of $(-4i + 4j)$ Ns on the ball. Find		
	(a)	the velocity of the ball immediately after the impact,	(3)
	(b)	the angle through which the ball is deflected as a result of the impact,	(2)
	(c)	the kinetic energy lost by the ball in the impact. (Total 9 mar	(4) :ks)
10.	A ten	nis ball of mass 0.2 kg is moving with velocity $(-10i)$ m s ⁻¹ when it is struck by a tennis	
	racke	t. Immediately after being struck, the ball has velocity $(15i + 15j)$ m s ⁻¹ . Find	
	(a)	the magnitude of the impulse exerted by the racket on the ball,	(4)
	(b)	the angle, to the nearest degree, between the vector i and the impulse exerted by the racket,	(2)
	(c)	the kinetic energy gained by the ball as a result of being struck.	
	(-)	(Total 8 mar	(2) :ks)

11.



A ball *B* of mass 0.4 kg is struck by a bat at a point *O* which is 1.2 m above horizontal ground. The unit vectors **i** and **j** are respectively horizontal and vertical. Immediately before being struck, *B* has velocity $(-20\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$. Immediately after being struck it has velocity $(15\mathbf{i} + 16\mathbf{j}) \text{ m s}^{-1}$.

After B has been struck, it moves freely under gravity and strikes the ground at the point A, as shown in the diagram above. The ball is modelled as a particle.

(a)	Calculate the magnitude of the impulse exerted by the bat on <i>B</i> .	(4)
(b)	By using the principle of conservation of energy, or otherwise, find the speed of B when it reaches A .	
		(6)
(c)	Calculate the angle which the velocity of B makes with the ground when B reaches A .	(4)
(d)	State two additional physical factors which could be taken into account in a refinement of the model of the situation which would make it more realistic.	
		(2)
	(Total 16 ma	rks)

1. (a)

$$0 \text{ m}_{s}^{-1}$$

$$4 \text{ m}_{s}^{-1}$$

$$R \xrightarrow{F} 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^{2}$$

$$P.E. lost = 0.6 \times g \times (12 \sin 30)$$
Change in energy = P.E. lost - K.E. gained
$$= 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^{2} = 0.6 \times g \times 12 \sin 30 - \frac{1}{2} \times 0.6 \times 4^{2}$$
MI AI AI AI
$$= 30.48$$
Work done against friction = 30 or 30.5 J
$$A1 = 4$$

$$R (\uparrow) \quad R = 0.6g \cos 30$$

$$B1$$

$$F = \frac{30.48}{12}$$

$$B1 ft$$

$$F = \mu R$$

$$\mu = \frac{30.48}{12 \times 0.6g \cos 30}$$

$$M1$$

$$\mu = 0.4987$$

$$\mu = 0.4997 \text{ or } 0.50$$

$$A1 = 4$$

$$I = mv - mu$$

$$= 0.5 \times 20i - 0.5(10i + 24j)$$

$$= 5i - 12j$$

$$A1$$

(b)

2.

(a)

[8]



(c) K.E.lost =
$$\frac{1}{2} \times 0.5 (10^2 + 24^2) - \frac{1}{2} \times 0.5 \times 20^2$$
 M1 A1

= 69 J A1 3

[9]

3. (a) Vertical motion:
$$v^2 = u^2 + 2as$$
 M1
 $(40 \sin \theta^2) = 2 \times g \times 12$ A1
 $(\sin \theta)^2 = \frac{2 \times g \times 12}{40^2} 40^2$
 $\theta = 22.54 = 22.5^\circ (\operatorname{accept} 23)$ A1 3

(b) Vert motion
$$P \to R$$
: $s = ut + \frac{1}{2}at^2$

$$-36 = 40\sin\theta t - \frac{g}{2}t^2$$
 M1

$$\frac{g}{2}t^2 - 40\sin\theta t - 36 = 0$$
 A1 A1

$$t = \frac{40\sin 22.54 + \sqrt{(40\sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$$

$$t = 4.694...$$
 A1

Horizontal P to R:
$$s = 40 \cos \theta t$$
M1 $= 173 \text{ m} (\text{or } 170 \text{ m})$ A1

(c) Using Energy:

$$\frac{1}{2}mv^{2} - \frac{1}{2}m \times 40^{2} = m \times g \times 36$$
 M1 A1

$$v^{2} = 2(9.8 \times 36 + \frac{1}{2} \times 40^{2})$$

$$v = 48.0....$$

$$v = 48 \text{ m s}^{-1} \text{ (accept 48.0)}$$
 A1 3
[12]



5

(b) Work Energy

First time at $X: \frac{1}{2}mv^2 = \frac{1}{2}m14^2$ Work done $= \mu \times R \times 2d = \frac{1}{8} \times 2g \times \frac{24}{25} \times 2d$ Return to $X: \frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8} \times 2g \times \frac{24}{25} \times 50$ M1A1 $v = 8.9 \text{ ms}^{-1}$ (accept 8.85 ms⁻¹) DM1A1 4 OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$

$$2a = 2g\sin\alpha - F_{\max} = 2g \times \frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$$
 M1A1

$$v^2 = (0+)2 \times a \times s = 8g$$
; $v = 8.9$ (accept 8.85 ms⁻¹) DM1;A1 [11]

5.	(a)	$\Delta KE = \frac{1}{2} \times 3.5 (12^2 - 8^2) (= 140) \text{ or KE at A, B correct separate}$	ly B1	
		$\Delta PE = 3.5 \times 9.8 \times 14 \sin 20^{\circ} (\approx 164.238)$ or PE at A, B correct sep	arately M1A1	
		$\Delta E = \Delta KE + \Delta PE \approx 304, 300$	DM1, A1	5

(b)	Using Work–Energy			
	$F_r = \mu \times 3.5g \cos 20^\circ$		M1A1	
	$304.238 = F_r \times 14$	ft their (a), F_r	M1A1ft	
	$304.238 \dots = \mu 3.5g \cos 20^{\circ} \times 14$			
	$\mu \approx 0.674, 0.67$		A1	



6. (a)		$R = 4g \cos \alpha = 16g / 5 \Longrightarrow F = 2 / 7 \times 16g / 5 $ M1 A		1	
		Work done = $F \times 2.5 = 22.4 \text{ J}$ or 22 J	Indep M1 A1	4	

(b)
$$\frac{1}{2} \times 4 \times u^2 = 22.4 + 4g \times 2.5 \times 3 / 5$$
 M1 A2,1,0 **f.t**.
 $\Rightarrow u \approx 6.37 \text{ m s}^{-1}$ or 6.4 ms⁻¹ A1 cao 4

(c)
$$\frac{1}{2} \times 4 \times v^2 = \frac{1}{2} \times 4 \times u^2 - 44.8$$

[OR $\frac{1}{2} \times 4 \times v^2 = 0 + 4g \times 2.5 \times 3/5 - 22.4$] M1 A2,1,0 **f.t.** 4
 $\Rightarrow v \approx 4.27 \text{ ms}^{-1}$ or 4.3 ms^{-1} A1
[12]

7. (a) Kinetic Energy =
$$\frac{1}{2} \times 3 \times 8^2 = 96$$
, J B1 B1 2

(b)	$F = \mu 3g$		B1	
	Work–Energy	$\mu 3g \times 12 = 96$	M1 A1ft	
	$\mu = 0.27 \text{ or } 0.272$		A1	4

$$a = \frac{8^2 - 0^2}{2 \times 12} = \frac{8}{3}$$

$$\mu 3g \qquad \qquad B1$$
N2L
$$\mu 3g = 3 \times \frac{8}{2} \qquad \qquad M1 A1$$

$$u = 0.27 \text{ or } 0.272$$

$$\mu = 0.27 \text{ or } 0.272$$
 A1 4

[9]

5

KE lost is $\frac{1}{2} \times 0.6 \times (10^2 - 9^2)$ (=5.7 J) (a) 8. **B**1 PE lost is $0.6 \times 9.8 \times 12 \sin 30^\circ$ (= 35.28 J) **B**1 Total loss in energy is 41.0 (J) M1 A1 4 accept 41

(b)
$$R = 0.6 \times 9.8 \times \cos 30^{\circ} (\approx 5.09)$$
 B1
WE $40.98 = \mu \times 0.6 \times 9.8 \times \cos 30^{\circ} \times 12$ M1 A1ft
 $\mu \approx 0.67 \text{ or } 0.671$ M1 A1 5
ft their (a)

Alternative for (b)

$$a = \frac{9^2 - 10^2}{2 \times 12} \left(= (-)\frac{19}{24} \right)$$
B1
awrt 0.79

 $mg\sin 30^\circ - \mu mg\cos 30^\circ = m\left(-\frac{19}{24}\right)$ N2L M1 A1ft $\mu \approx$

$$\approx 0.67 \text{ or } 0.671 \qquad \qquad \text{M1 A1}$$
ft their a

9. (a)
$$I = mv - mu$$

 $-4i + 4j = 0.2v - 0.2 \times 30i$ M1 A1
 $v = 10i + 20j \text{ (m s}^{-1})$ A1 3

(b)
$$\tan \theta = \frac{20}{10}$$
 M1
 $\theta = 63.4^{\circ}$ A1 2

(c) Final K.E. =
$$\frac{1}{2} \times 0.2 \times (10^2 + 20^2)$$
 (= 50) ft their **v** M1 A1ft
K.E. lost = $\frac{1}{2} \times 0.2 \times 30^2 - \frac{1}{2} \times 0.2 \times (10^2 + 20^2)$ M1
= 40 (J) cao A1 4
[9]

10. (a)
$$I = 0.2[(15i + 15j) - (-10i)]$$

= 5i + 3j
 $|I| = \sqrt{(5^2 + 3^2)} = \sqrt{34} = 5.8 \text{ Ns}$
M1 A1 4

(b)
$$\theta = \frac{3}{5} \Rightarrow \theta = 31^{\circ}$$
 (nearest degree) M1 A1 ft 2

(c) KE Gain =
$$\frac{1}{2} \times 0.2[(15^2 + 15^2) - 10^2)] = 35 \text{ J}$$
 M1 A1 2

[8]

4

11. (a)
$$\mathbf{I} = 0.4(15\mathbf{i} + 16\mathbf{j} + 20\mathbf{i} - 4\mathbf{j}) (= 0.4(35\mathbf{i} + 12\mathbf{j}) = 14\mathbf{i} + 4.8\mathbf{j})$$
 M1
 $|\mathbf{I}| = \sqrt{(14^2 + 4.8^2)} \text{ or } 0.4\sqrt{(35^2 + 12^2)}$ M1 A1
M1 for any magnitude
 $= 14.8 \text{ (Ns)}$ A1

(b) Initial K.E. =
$$\frac{1}{2}m(15^2 + 16^2) (= 240.5m = 96.2 \text{ J})$$
 M1
 $\frac{1}{2}mv^2 = \frac{1}{2}m(15^2 + 16^2) = m \times 9.8 \times 1.2$ M1 A2, 1,0

$$-l$$
 each incorrect term
 $v^2 = 504.52$

$$v^2 = 504.52$$
 M1
 $v = 22 \text{ (m s}^{-1})$
accept 22.5 A1 6

[16]

(c)	$\arccos \frac{15}{22.5} = 48^{\circ}$	M1 A1 A1 A1	4	
	accept 48.1 °			
(d)	Air resistance	B1. B1	2	

(u)	An resistance	D1, D1	4
	Wind (problem not 2 dimensional)		
	Rotation of ball (ball is not a particle)		
	any 2		

Alt (b)			
Resolve \uparrow with 16 and 9.8	M1		
(1) $v_y^2 = 16^2 + 2 \times (-9.8) \times (-1.2)$	M1 A1		
$(v_y^2 = 279.52, v_y \approx 16.7)$			
$v^2 = 15^2 + 279.52$	M1 A1		
$v = 22 \text{ (m s}^{-1}\text{)}$	A1	6	
accept 22.5			

arctan	$\frac{16.7}{15} = 48^{\circ}$	M1 A1 A1 A1	4	
	10			[10]

1. This question proved to be straightforward for well-prepared candidates.

In part (a) it was pleasing to see many candidates tackling this using the work-energy method, and there was less evidence this year of candidates double counting by including both the change in GPE and the work done against the weight, but candidates sometimes confused work done with just potential energy lost, or just kinetic energy gained. The alternative method using *suvat* to find the acceleration and then using F = ma was also common. In the final answers there was considerable confusion between work done against friction and the frictional force. Many lost the final A mark by leaving the answer as 30.48 despite having used g = 9.8.

In part (b) candidates frequently did not make the connection with part (a) and proceeded to start again from scratch. In this case, a common but expensive error was to omit the component of the weight from their equation of motion.

2. This question was well attempted by a majority of candidates.

In part (a) the most common incorrect answer was a sign error leading to an impulse of $(5\mathbf{i} + 12\mathbf{j})$ Ns rather than $(5\mathbf{i} - 12\mathbf{j})$. Some candidates failed to apply the impulse formula correctly, adding momentum rather than subtracting. Many students forgot to calculate the magnitude of their impulse. A few candidates started by finding the initial and final speeds of the ball and ignored the two dimensional nature of the problem, never producing a vector equation for the impulse or appropriate work using trigonometry.

In aprt (b) a common error was to find the angle for the initial velocity rather than the impulse. A minority of candidates were confused over which angle was required or made a trigonometric

error, using the ratio $\frac{5}{12}$ rather than $\frac{12}{5}$ to find the angle.

For part (c) although there were many completely correct solutions, some candidates were unable to cope with using vectors to find speed and hence Kinetic Energy. The most common errors were, for example, to find $\sqrt{10^2 + 24^2}$ and then forget to square it or to attempt to square the vector velocity, treating $(10\mathbf{i} + 24\mathbf{j})^2$ as an algebraic expression and retaining \mathbf{i} and \mathbf{j} components in the answer.

3. There were many confident solutions to this question, but over specification of answers following the use of g = 9.8 was a common problem, causing many candidates to throw away a mark.

In part (a) many students used correct methods for calculating the angle, although 22.54 rather than 22.5 was common. Very few went wrong here, though some took longer routes than necessary, failing to spot that they could use $v^2 = u^2 + 2as$ to obtain the angle in one step, and there were a few who attempted to use distances to find the angle.

In part (b) the quickest method here was to use a displacement of -36 in an equation to find time and then use this time in a horizontal equation to find displacement. Some took this in two stages: time to the highest point and then time to the bottom. It was pleasing to find very few resolution and both methods were used correctly in many solutions. The most common error was to consider only part of the flight and then use an incorrect time to find the horizontal displacement. The over-specified answer 173.4 rather than 173 was common.

In part (c) those candidates who used conservation of energy were usually successful but the most common method was to find horizontal and vertical components of velocity and hence find the speed using Pythagoras' Theorem. Unfortunately, those who used this method often

found only the vertical component of the velocity and lost all the marks here.

- 4. In this question a significant minority tried to find the acceleration in part (a), despite being instructed to use work and energy. For those candidates using the required method, the most common error was to include the work done against gravity with the work done against the friction but then also include the change in gravitational potential energy, thus double counting. Although many answered successfully, there were some who rather "fudged" the arrival at the given answer. A common error was to assume that the vertical height gained was 7, and this lead inevitably to the correct answer without the necessary algebraic treatment of the situation. The most common approach to part (b) was to revert to the use of F = ma, rather than sticking to the work-energy principle, for which they had already done most of the work in part (a). Many who attempted a work-energy approach forgot to include the work done against friction in this part, even though they had used it in part (a). Very few candidates used energy and point X as the initial and final position to find the answer.
- 5. (a) Many candidates lost marks here. This was usually because they found only the loss in kinetic energy rather than using both the kinetic energy and the potential energy terms
 - (b) Those candidates who chose to use their answer to part (a) and use the work-energy principle tended to be successful in finding a value for μ , but a significant number did not realise the connection between parts (a) and (b). Those who chose to use constant acceleration and then F = ma generally found μ correctly. The most common errors were to omit the weight component in their equation for F = ma or to confuse the two methods and include extra energy terms in their energy equation.

Many candidates lost marks through giving answers to inappropriate accuracy following the use of an approximate value for *g*.

- 6. Many students were unable to apply the 'work-energy' principle efficiently or accurately. In part (a) there was sometimes confusion between work done against friction and work done against gravity and often the weight component was thought to be part of the friction force. Relatively few candidates realised that they could use their answer to part (a) in their solution to part (b) and either started again or else just ignored the friction altogether. Some just ignored the instruction to "use the work-energy principle", and scored no marks. The final part was usually done better, as candidates could use a force-acceleration method rather than work-energy and there were many correct solutions.
- 7. This was generally well answered almost every candidate identified Joules as the units of energy in the first part and most found the correct numerical value. The force-acceleration method was the most popular approach in part (b). There were some problems with accuracy and signs.

- 8. Energy and the work-energy principle are an area of weakness for many and, in part (a), it was quite common for candidates to find only the change in kinetic energy, ignoring the change in potential energy. No method was specified for part (b) and candidates were roughly evenly divided between those who used the work-energy principle, utilising their answer to part (a) and those who, essentially started again, using Newton's second law. If they had an incorrect answer to part (a), those using work-energy could gain 4 of the 5 marks in part (b). Errors of sign were often seen in the solutions of those who used Newton's second law. This often arose through uncertainty about the direction of the acceleration. A substantial minority of candidates treated the forces parallel to the plane as being in equilibrium, assuming that there was no acceleration.
- 9. Part (a) was well done, although a minority changed the sign of the velocity from (30i) m s⁻¹ to -(30i) m s⁻¹, thinking that they were taking a "rebound" into account, not understanding that the direction was implied by the vector itself. In part (b) there was some confusion between the angle of deflection and the angle the impulse made with i. Part (c) was quickly done by those who knew how to find the kinetic energy associated with a velocity given in vector form but many could not do this.
- **10.** (a) Many found the difference in the magnitudes of the momentum vectors instead of the other way round. Of those who knew the method for finding the impulse, many then forgot to find the magnitude of it.
 - (b) There was a good level of success with finding the angle although some forgot to round it to the nearest degree. The most common error was to get the tan upside down.
 - (c) The vectors here caused some confusion for the weaker candidates, with final answers often appearing as vectors.
- 11. As is appropriate for the last question, this proved the most difficult on the paper. Those who were confident with vector notation often produced accurate and concise solutions. Many lost marks in part (a) by not seeing that the question asked for the *magnitude* of the impulse. Others found m(|v|-|u|). Part (b) caused a lot of confusion among candidates. They were often uncertain whether they should be using vectors, scalars, component velocities or the resultant and often ended up using unacceptable combinations of these. It was common to see $v^2 = u^2 2gs$ being used with $u^2 = 15^2 + 16^2$ and, also, for the final vertical velocity component to be given as the required speed. Part (c) caused less confusion and most candidates were aware that they had to draw a triangle and use trigonometry to find the required angle. At the relatively low speeds on this question, it was thought that sensible answers to part (d) were air resistance and the possibility of cross winds. A number of candidates mentioned that the initial impact between the bat and ball would probably impart some spin or rotation to the ball and that a full analysis of the motion would take this into account and not consider the ball as a particle. This was accepted.